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# Approximate Hemispherical Radiative Properties

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## Introduction

THE several investigations of radiative properties of surfaces that have appeared in the literature[1-3] have dealt primarily with emissive characteristics and, in particular, with the estimates of the monospectral hemispherical emittance. These estimates depend on the optical constants of the emitting material, that is, the real portion of the index of refraction $n_\lambda$, the imaginary portion of the index of refraction, $k_\lambda$ ($N_\lambda = n_\lambda(1-ik_\lambda)$), and the receiving material $N_0$. The subscript $\lambda$ indicates wavelength dependence. It is usually conceded that the hemispherical properties are more easily estimated when the normal emittance is known or can be determined. This is a convenience which stems from the fact that emittance measurements are often performed at or near normal and charts of the ratio of the hemispherical emittance to the normal emittance are available.[2,3] These charts have been made available after long numerical integrations, but only for a few values of $n_\lambda$ and $k_\lambda$.

Approximate closed-form expressions of the monospectral hemispherical emittance have been obtained under the condition that $n_\lambda^2(1+k_\lambda^2)$ be very large compared to unity. The resulting approximation is essentially exact in the cases where $n_\lambda$ and $k_\lambda$ are greater than unity (typical of metals in the infrared regions). In the case where the refractive index is less than unity, a large error results. Hering[3] has set forth various accuracy criteria for $n_\lambda$ and $k_\lambda$.

The purpose of this Note is to indicate another approximation for monospectral hemispherical emittance determined from the monospectral hemispherical reflectance (i.e., $\epsilon_h(\lambda) = 1 - \rho_h(\lambda)$).

## Development

The expressions from electromagnetic theory that represent the monospectral, specular reflectance of nonmagnetic material polarized perpendicular to, $\rho_\perp(\phi,\lambda)$, and parallel to, $\rho_\parallel(\phi,\lambda)$, the plane of incidence can be stated as[4]:

$$\rho_\perp(\phi,\lambda) = \frac{a^2 + b^2 - 2a\cos\phi + \cos^2\phi}{a^2 + b^2 + 2a\cos\phi + \cos^2\phi} \tag{1}$$

Received Oct. 27, 1978. 

Index category: Radiation and Radiative Heat Transfer.

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$$\rho_\parallel(\phi,\lambda) = \frac{\rho_\perp(\phi,\lambda)\,(a^2 + b^2 - 2a\sin\phi\tan\phi + \sin^2\phi\tan^2\phi)}{(a^2 + b^2 + 2a\sin\phi\tan\phi + \sin^2\phi\tan^2\phi)} \tag{2}$$

where

$$2a^2 = \{[n_\lambda^2(1-k_\lambda^2) - \sin^2\phi]^2 + 4n_\lambda^4 k_\lambda^2\}^{1/2} + n_\lambda^2(1-k_\lambda^2) - No^2\sin^2\phi \tag{3}$$

and

$$2b^2 = \{[n_\lambda^2(1-k_\lambda^2) - \sin^2\phi]^2 + 4n_\lambda^4 k_\lambda^2\}^{1/2} - n_\lambda^2(1-k_\lambda^2) + No^2\sin^2\phi \tag{4}$$

In these equations, $\phi$ is the angle of incidence. For the case of natural or equally polarized light, the angular monospectral reflectance is the average of these polarized components:

$$\rho(\phi,\lambda) = \frac{\rho_\perp(\phi,\lambda) + \rho_\parallel(\phi,\lambda)}{2} \tag{5}$$

The monospectral hemispherical reflectance $\rho_h(\lambda)$ as defined by Sparrow and Cess[5] and Siegal and Howell[6] can be obtained by integrating this angular reflectance:

$$\rho_h(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} \frac{\rho(\phi,\lambda)}{\pi}\cos\phi\sin\phi\,d\phi\,d\theta \tag{6}$$

or

$$\rho_h(\lambda) = \int_0^{2\pi}\int_0^{1} \frac{\rho(\mu,\lambda)}{\pi}\mu\,d\mu\,d\theta \tag{7}$$

This hemispherical reflectance and the corresponding monospectral hemispherical emittance are commonly used when radiant energy exchange is calculated. Judd[7] has calculated this hemispherical reflectance according to approximate formulas for internally and externally incident, perfectly diffuse monospectral light and compared his values to the values of reflectance at perpendicular incidence for the special case where $k$ is equal to 0.0, i.e., dielectric material.

The total hemispherical reflectance is determined by integrating the monospectral hemispherical reflectance and the source function $H_\lambda$, per the following equation:

$$\rho_h = \int_0^\infty \rho_\lambda H_\lambda d\lambda \Big/ \int_0^\infty H\lambda\, d\lambda \tag{8}$$

This source function $H_\lambda$, represents the spectral distribution of the source irradiating the material of interest ($n_\lambda, k_\lambda$).

## Results

Equation (7) was numerically integrated for a large variety of optical constants ($n_\lambda, k_\lambda$) for $N_0 = 1$. When these results were plotted with the results obtained from the Fresnel reflectance equation [i.e., Eqs. (1-5)] for $\phi = 60$ deg, an interesting pattern emerged. Figures 1-3 are typical examples. Included on these figures are dashed lines indicating lines of 5% and 10% error, as well as a solid line which indicates perfect agreement between $\rho_h(\lambda)$ and $\rho$(60 deg, $\lambda$). Using this type of information, Fig. 4 was deduced. Figure 4 presents values of $n_\lambda$ and $k_\lambda$ for which $\rho$(60 deg, $\lambda$) may be substituted for $\rho_h(\lambda)$ with expected errors of greater than 10%, between 5% and 10%, and less than 5%.

If the optical constants are known for a material, a fairly accurate estimate of the hemispherical reflectance value can be obtained merely by applying the Fresnel equations at the appropriate angle; that is, from Eq. (6),

$$\rho_h(\lambda) \doteq \rho(60\text{ deg},\lambda)$$

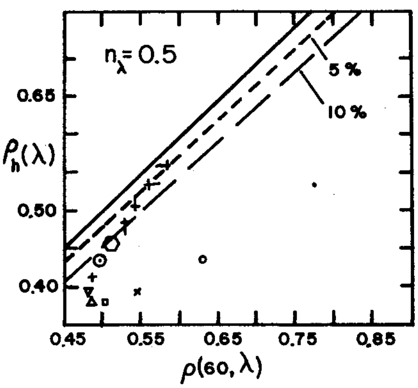


**Fig. 1 Monospectral hemispherical reflectance vs Fresnel specular reflectance ($\phi = 60$ deg) for $n_\lambda = 0.5$. $\bullet k_\lambda = 0.2$, $\circ\ k_\lambda = 0.4$, $\times\ k_\mu = 0.6$, $\square\ k_\lambda = 0.8$, $\triangle\ k_\lambda = 1.0$, $\triangledown\ k_\lambda = 1.2$, $+k_\lambda = 1.4$, $\odot\ k_\lambda = 1.6$, $\bigcirc k_\lambda = 1.8$, $|k_\lambda = 2.0$, $|k_\lambda = 2.2$, $-k_\lambda = 2.4$, $-k_\lambda = 2.6$.**

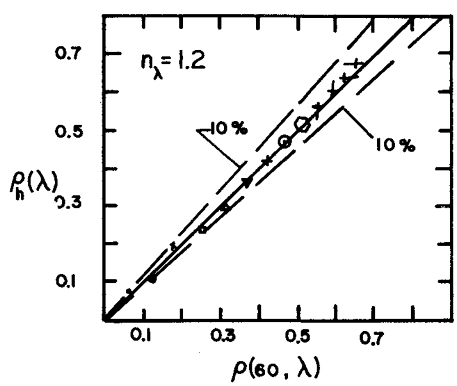


**Fig. 2 Monospectral hemispherical reflectance vs Fresnel specular reflectance ($\phi = 60$ deg) for $n_\lambda = 1.2$ (same legend as Fig. 1).**

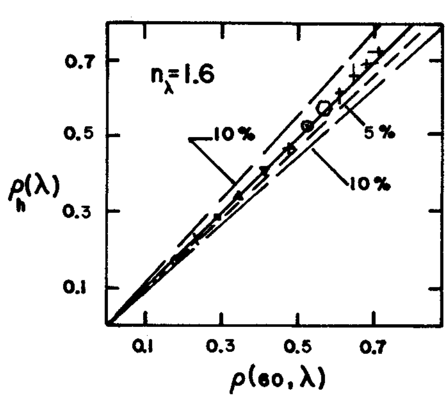


**Fig. 3 Monospectral hemispherical reflectance vs Fresnel specular reflectance ($\phi = 60$ deg) for $n\lambda = 1.6$ (same legend as Fig. 1).**

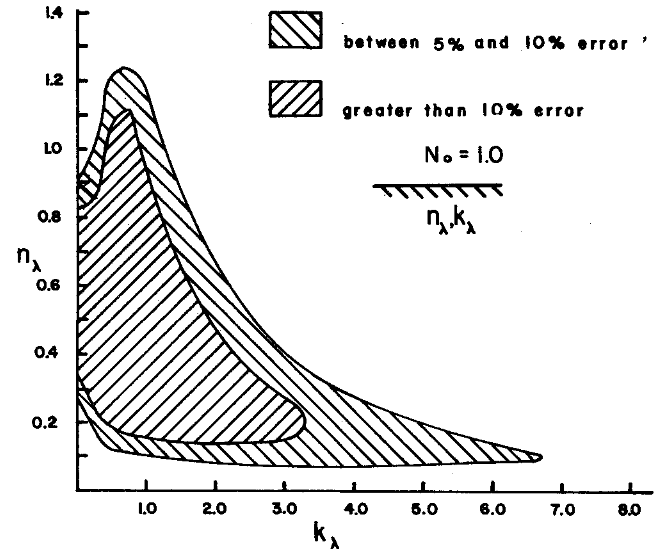


**Fig. 4 Illustration of values of $n_\lambda$ and $k_\lambda$ resulting in more than 10% error between 5% and 10% error, and less than 5% error when $\rho$(60 deg, $\lambda$) is compared to $\rho_h(\lambda)$.**

and thus Eq. (8) may be approximated as:

$$\rho_h = \int_0^\infty \rho(60\ \mathrm{deg},\lambda) H_\lambda \mathrm{d}\lambda \Big/ \int_0^\infty H_\lambda \mathrm{d}\lambda$$

where $\rho(60\ \mathrm{deg},\lambda)$ may be represented by:

$$\rho(60\ \mathrm{deg},\lambda) = 1 - 4a\left\{ \frac{(21+12a+20a^2)+20b^2}{[4b^2+(3+2a)^2][4b^2+(1+2a)^2]} \right\}$$

## Acknowledgment

The author wishes to thank C. J. Warren for her efforts in this project.

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